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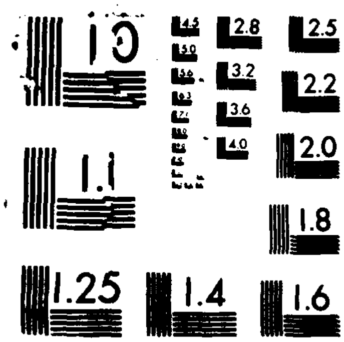
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*Benoit Montreuil
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ABSTRACT

Facility layout provides a fertile area for the application of optimization models. There is a major cost associated with constructing or reconfiguring a facility. In addition, the number of layout possibilities is so large that human designers frequently have difficulty in generating good layouts. A cut tree is proposed here as the basis for a family of facility layouts. It has some very attractive properties when used as part of an interactive system. The cut tree has proven to be both easy to use and very powerful in aiding designers to generate quality layouts.

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I. INTRODUCTION

It is generally recognized that computer models cannot provide adequate solutions to facility layout problems without considerable assistance from human designers. This results from the mathematical difficulty of the problems being addressed and the fact that the design objectives and constraints cannot in general be precisely defined. However, computer models can be extremely valuable in the design process by providing "design skeletons" which give human designers a framework from which to start the design process. The availability of computer graphics has greatly enhanced the ability to integrate sophisticated models for generating design skeletons into interactive design systems.

Two of the most intuitive structures on which to base design skeletons are adjacency graphs and material flow graphs. In an adjacency graph, nodes represent departments and links represent the pairs of departments which are desired to be adjacent. In a material flow graph, nodes represent input/output stations for departments and links represent the flow between pairs of input/output stations.

The Systematic Layout planning approach (Muther [1974]), the planar graph approach (Seppanen and Moore [1979] and Foulds [1983]) and the matching approach (Montreuil et. al. [1987]) all

construct design skeletons based on adjacency graphs. While these approaches have proven effective for a wide range of layout problems, adjacency graphs have two major shortcomings for use as the basis for a design skeleton when material flows are a major factor. First, adjacency graphs are limited to consideration of only those flows between adjacent departments. Second, adjacency graphs completely ignore the flow structure (i.e., the actual paths over which the material flows).

An alternative to working with the adjacency graph is to work with the material flow graph. If the material flow graph is sparse (i.e., there are relatively few pairs of departments which have flow between them), it is frequently possible to consider the entire graph as a design skeleton and evolve it into the flow structure (i.e., aisle structure or conveyor network) along which the material will move. When the material flow graph is dense (i.e., many pairs of departments have flow between them), it is very difficult to manipulate the graph directly. In this case some modelling tools are required to help the designer extract a good design skeleton from the flow graph.

II. TREE SKELETONS

There are a number of characteristics that are desirable in a design skeleton.

1. It should be easy for the designer to understand and manipulate.

2. It should provide the designer with guidance for the layout while leaving considerable flexibility to take advantage of experience and intuition.

3. It should lead to an efficient flow structure in terms of aisle space and flow control.

4. It should provide a theoretical reference point for the designer.

An attractive class of design skeletons with these properties is obtained by generalization of the "spine" layout concept discussed by Thompkins [1980]. When viewed as a graph, a spine structure is a special case of a "spanning tree" (i.e., an acyclic connected graph where nodes represent input/output stations and links represent the flow structure or aisles which connect the stations). Figure 1 illustrates and aisle structure which is a tree.

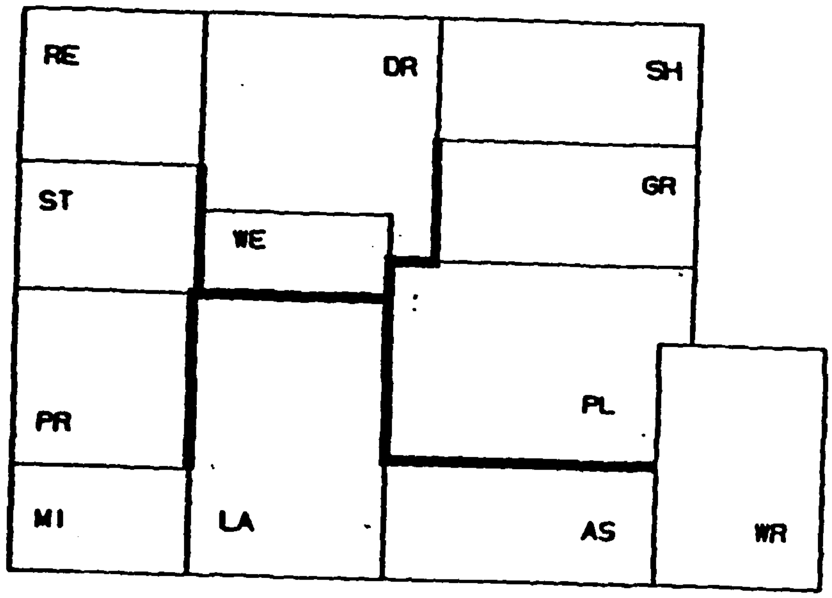

Figure 1: Illustration of an aisle structure which is a tree.

With regard to the desirable characteristics listed above: (a) trees are among the simplest graphs to understand and to manipulate manually; (b) they provide the designer with a lot of flexibility in generating the layout since a particular tree can be drawn in a variety of ways each leading to a different layout; (c) they provide a very simple flow control structure since there is a unique path between each pair of stations; and (d) they tend to minimize the amount of aisle space required in the sense that none of the links of a spanning tree can be removed without disconnecting the graph. The issue of providing a theoretical reference point will be addressed later.

The use of a spanning tree as a design skeleton for a layout was proposed by Carrie [1973] and discussed further in Moore and Carrie [1975]. They suggest using the maximum spanning tree based on direct flows between departments. The maximum spanning tree has the advantage that it can be very efficiently determined. However, it does not provide the desired theoretical reference point and more importantly, it is similar to the adjacency graph approaches discussed above in that it does not consider all of the flows in arriving at a design skeleton. It considers only n-1 (where n is the number of input/output stations) of the largest flows. This is not meant to downplay the value of using a maximum spanning tree as a design skeleton. It has the important advantage that the maximum spanning tree in

a graph can be determined by hand for very large graphs. This can be critical if more sophisticated procedures are not available.

However, a spanning tree which is in many ways more appealing than the maximum spanning tree for use as a design skeleton is a "cut tree". The concept of a cut tree was introduced by Gomory and Hu [1961]. A cut tree for a graph is a spanning tree where the arc of minimum weight on the unique path separating two nodes corresponds to the minimum cut separating the two nodes in the original graph. This concept is explained in the next section.

II. CUT TREES

To illustrate the concept of a cut tree, consider the interstation flow exchanges given in Table 1. The flow exchanges represent the average amounts of material movement between stations.

DEPARTMENT	CODE	AREA
Receiving	RE	1800
Milling	MI	1200
Presses	PR	2000
Lathes	LA	3600
Drills	DR	3200
Welding	WE	1000
Plating	PL	3600
Grinding	GR	2000
Assembly	AS	2100
Warehouse	WA	2600
Shipping	SH	2000
Stores	ST	1500

Table 1. Department codes and areas for example problem.

The graph in Figure 2 has a node representing the input/output station for each department in Table 1. Each link represents the flow exchange between the corresponding departments (i.e., there are 30 trips per hour between receiving and the lathes). The tree in Figure 3 is a minimum cut tree for the graph in Figure 2. For a discussion of the minimum cut tree algorithm see Hu [1970].

Note in Figure 3 that there is a unique path between each pair of nodes. For example, the unique path between PR and GR is

PR-LA-PL-GR. The minimum link value along the path is the minimum of (90,100,110). Gomory and Hu [1961] show that the minimum cut separating PR and GR is found by breaking the PR-LA link. This means that a minimum cut in the graph in Figure 1 separating PR and GR is defined by putting RE, ST, MI, and PR on one side of the cut and all of the other nodes on the other side. The value of this cut (i.e., the sum of the link numbers in the cut is 90).

It should be noted that the problem of finding a minimum cut tree with 100 nodes (i.e., 100 departments in the layout) would correspond to a large layout problem. Such cut tree problems can be solved in a few minutes on an IBM PC/AT. Hence, for most practical layout problems the corresponding cut tree problem can be easily solved on a PC.

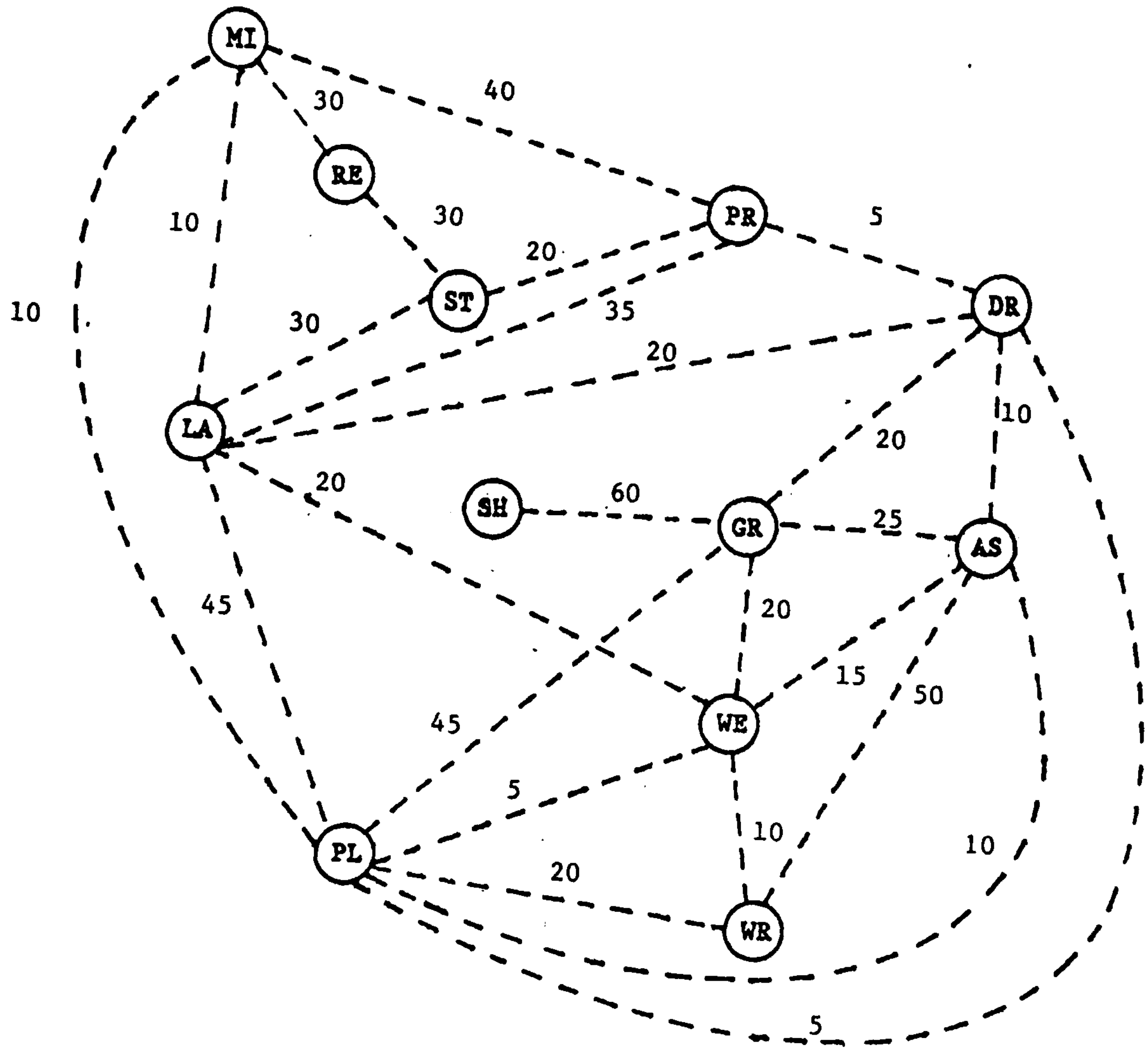

Figure 2: The material flow graph corresponding to the data in Table 1.

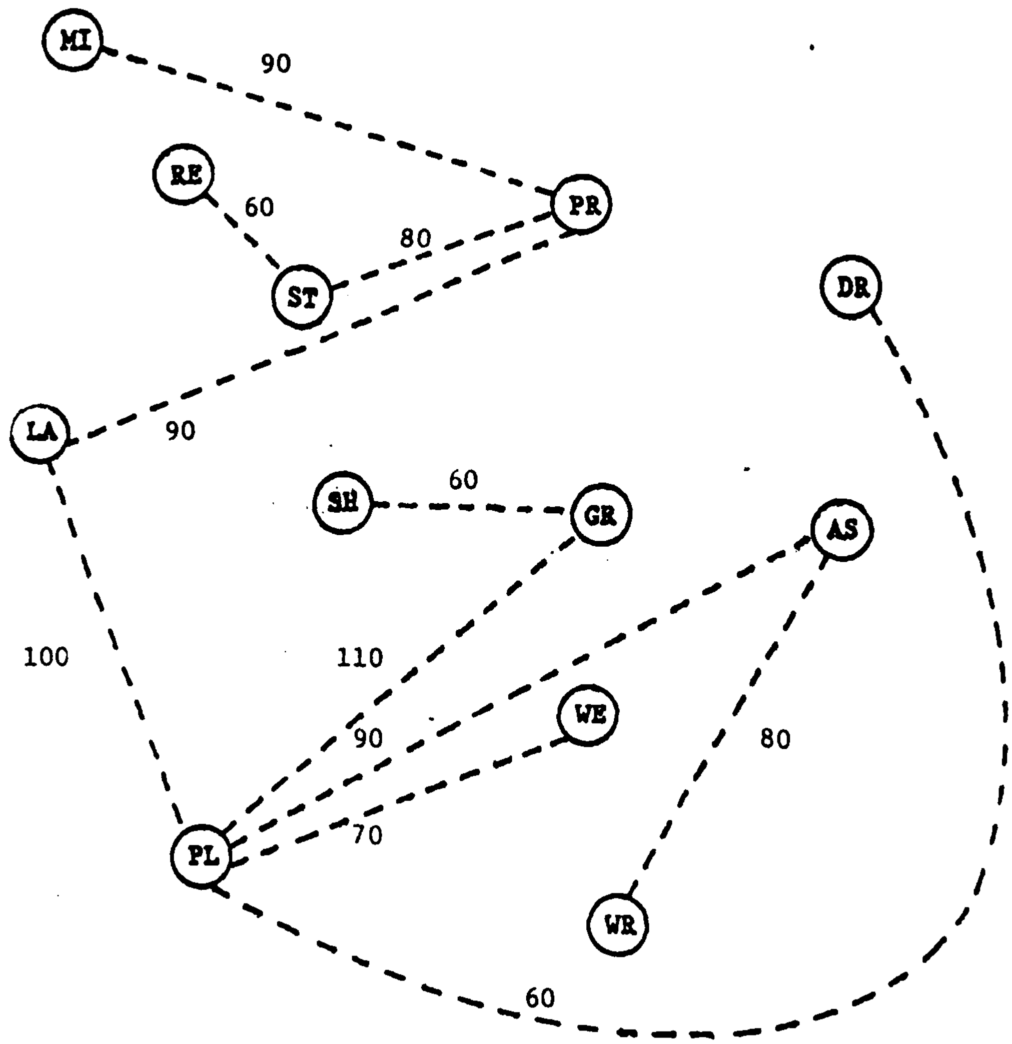

Figure 3: A cut tree for the material flow graph in Figure 2.

III. CUT TREE INTERPRETATION

The cut tree has some very useful characteristics when used as a design skeleton for a layout.

Property 1: If you want to partition the departments into two nonempty sets so that the flow between the two sets is minimized, the cut tree indicates the optimum partition.

To illustrate property 1, suppose that you want to put the plating (PL) and grinding (GR) departments on different floors, then by putting RE, ST, MI, and PR on the same floor as plating and the remaining departments on the same floor as grinding, the average flow between floors will be minimized. Furthermore, the average flow between floors will be 90 trips per hour. This follows since this partition corresponds to the minimum cut separating PL and GR in Figure 2.

Property 1 indicates a fundamental difference between the cut tree and the other graph theoretic models proposed to date for obtaining a design skeleton. The cut tree focuses on which departments should be separated while the other models focus on which departments should be made adjacent to each other.

Property 2: The numbers on the links of the cut tree indicate the average amount of flow which would cross each link if the

tree is used as the linking structure.

This provides the designer with some very valuable insights with regard to the cost of increasing the length of the aisles. For example, if the only connecting aisles among departments were those represented by the tree in Figure 3, the average flow on the aisle connecting PR1 and LA1 would be 90. Hence, increasing the length of this aisle will increase the average travel by 90 times the increase.

<u>Property 3:</u> If the aisle structure for the layout is restricted to be a tree with all aisle segments the same length, then the cut tree provides the aisle structure which minimizes the number of trips times the distance travelled.

Property 3 follows from a result by Adolphson and Hu [1973]. While we would not generally have equal length aisle segments, the cut tree provides a theoretical reference point for construction of the layout.

IV. LAYOUT CONSTRUCTION

Given a material flow graph (Figure 2), the algorithm of Gomory and Hu [1961] can be applied to efficiently generate a cut tree (Figure 3). The cut tree provides the designer with insights regarding the layout but it <u>does not</u> solve the layout problem for him.

The next step is for the designer to manipulate the cut tree so that the nodes are positioned at "approximate" locations of the input/output stations for the corresponding departments. This is perhaps the most difficult step in the process. Based on the properties discussed in the previous section, the designer is motivated to position the nodes as compactly as possible while allowing space to position all of the departments. However, the actual positioning of the nodes depends on the particular problem being addressed along with the insight and experience of the designer. The nodes in the cut tree in Figure 3 must be positioned inside the constraining walls, stairs, etc. as is shown in Figure 4. This frequently takes considerable effort on the part of the designer but is greatly facilitated by the use of computer graphics. Note that there is no obvious "best" positioning of the nodes. Acceptable positioning is dependent on the problem and on the objectives of the analyst.

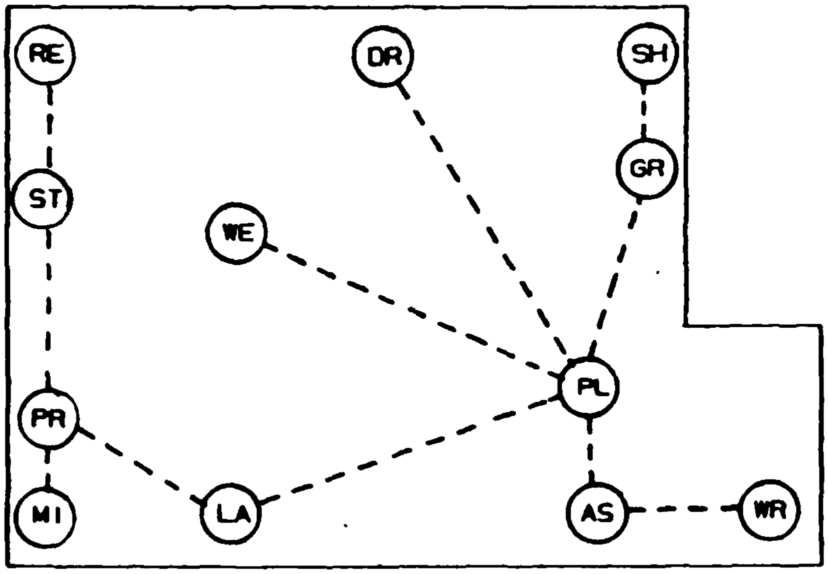

Figure 4: A positioning of the cut tree in Figure 3 inside the building boundaries.

Once the input/output stations have been tentatively positioned, the appropriate space must be assigned to each department. For the example in Figure 4, this is shown in Figure 5. Again, there is no obvious "best" assignment of space to departments. Different analysts can, and probably will, arrive at totally different assignments of space for a given positioning of the nodes.

Finally, the cut tree must be "conformed" to correspond to the actual aisles in the building. For the example in Figure 5, this is shown in Figure 6. This is required to eliminate excessive aisles and to eliminate aisles at odd angles.

Obviously, the layout shown in Figure 6 is only one of many which could have been generated based on the cut tree in Figure 4. The number of possible layouts is limited only by the creativity of the designer. Since the objective of the designer is "almost always" to generate a "good" design rather than a theoretically optimum design, this flexibility should be viewed as an attractive characteristic of the tree structure as a design skeleton.

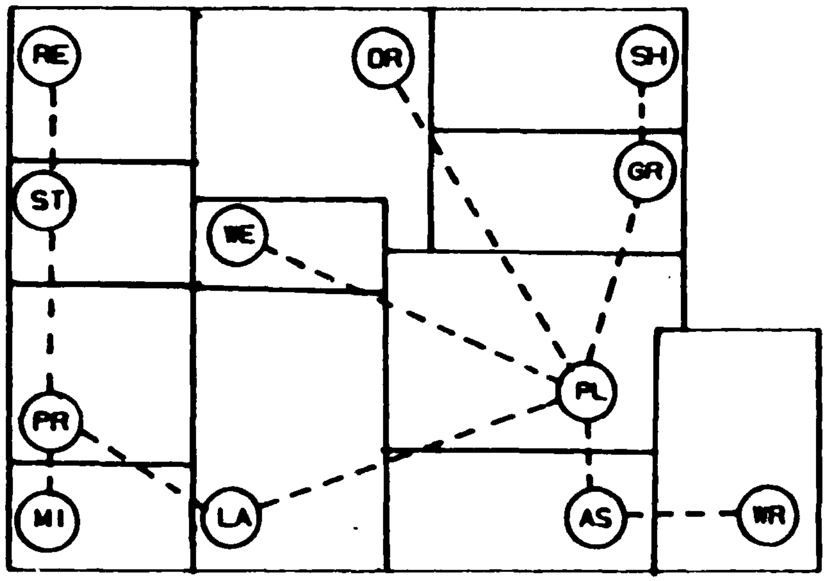

Figure 5: An assignment of the space requirements from Table 1 to the tree position of Figure 4.

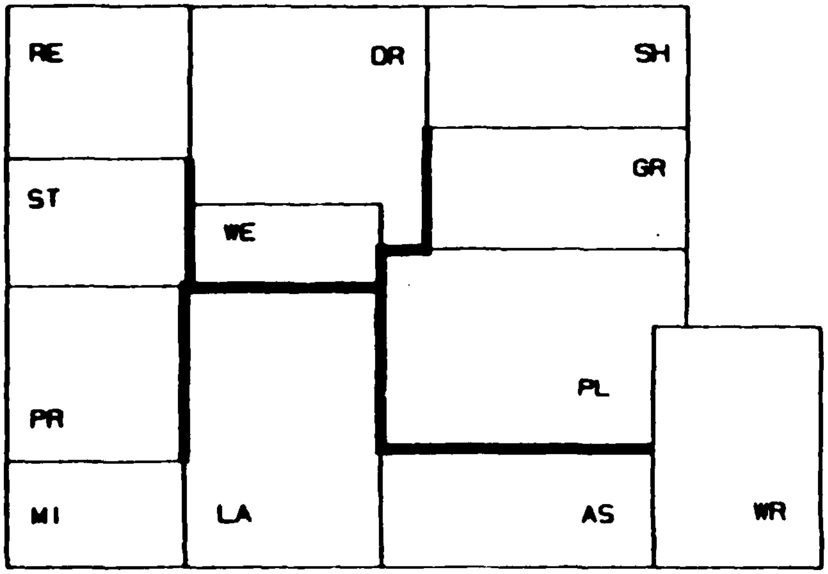

Figure 5: A layout after the tree structure in Figure 4 has been "conformed" to the department structure.

V. SUMMARY

The cut tree seems to be a valuable tool for the facility designer to add to his collection. We have utilized the cut tree on a number of actual layout problems and have been pleased with its value in providing new insights to aid in the layout process. As with all layout tools, the value of the cut tree as an aid to layout is very difficult to establish in any rigorous fashion. However, it does have a more substantial theoretical basis than many of the tools currently in use. In addition, it is very easy to understand and use and provides the designer with the flexibility to use his insight and experience to work with the tool in generating good layouts.

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